WORKSHEET 5

Date: 10/11/2021

Name:

1. (a) Let a, b be rationals and x irrational. Show that if $\frac{x+a}{x+b}$ is rational, then $a = b$

   (b) Let x, y be rationals such that $\frac{x^2+x+\sqrt{2}}{y^2+y+\sqrt{2}}$ is also rational. Prove that either $x = y$ or $x + y = -1$

2. Consider a rectangle with positive sides $a, b \in \mathbb{R}$. Is it possible to find values a,b such that the perimeter is rational but the area is irrational?

3. Determine whether the following statements are true or false. If they are true, prove them. If they are false, give a counterexample.

   (a) If x and y are both irrational numbers, then their sum and product are also irrational.

   (b) Every nonzero rational number is equal to a product of two irrational numbers.

4. Prove $\sqrt[3]{2}$ is irrational.

5. Prove that if x is irrational and $x \geq 0$, then $\sqrt{x}$ is irrational.